RAN-2103000206020034

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T. Y. B. Sc. (Sem - VI) Examination April - 2023

Mathematics : Paper - MTH - 604

Real Analysis - IV

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T. Y. B. Sc. (Sem - VI)**

Name of the Subject :

☛ **Mathematics : Paper - MTH - 604 Real Analysis - IV**

Subject Code No.: **2103000206020034**

Seat No.:

Student's Signature

(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

Q. 1. Answer any Five the following as directed : (10)

(1) Prove that every finite set in any metric space is closed.

(2) Prove that $\{2022\} \cup [20, 23]$ is a closed subset of the metric space $\mathbb{R}^1$.

(3) Prove that $\left\{\frac{22}{23}, \frac{23}{22}\right\}$ is a totally bounded subset of the metric space $\mathbb{R}^1$.

(4) (i) Give one example of a bounded subset of the metric space $\mathbb{R}_d$ which is not totally bounded ;

(ii) Give an example of a totally bounded subset of the metric space $\mathbb{R}_d$ which is not connected.

(5) Define Complete Metric Space and Fixed - Point.

(6) Justify : $\{20, 22, 23\}$ is not a complete subset of the metric space $\mathbb{R}^1$.

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(7) (i) Give an example of a subset of $\mathbb{R}_d$ which is compact as well as connected;

(ii) Give an example of a connected subset of $\mathbb{R}^1$ which is not compact.

(8) State the Heine-Borel Property.

Q. 2. Answer any Two of the following : (10)

(a) Let E be the subset of a metric space $\langle M, \rho \rangle$. Prove that $\overline{E}$; the closure of E; is closed.

(b) Prove that a union of two closed sets in any metric space is closed. What do you say about an infinite union of closed subsets in a metric space?

(c) (i) Justify : (2022, 2023) is not a closed subset of the metric space $\langle (2022, 2023), |\cdot| \rangle$;

(ii) Prove that the complement of an open set in a metric space is closed.

Q. 3. Answer any Two of the following : (10)

(a) Define Connected Set. Prove that [0, 2023] is not connected subset of the metric space $\mathbb{R}_d$.

(b) If A_1 and A_2 are connected subsets of a metric space $\langle M, \rho \rangle$ and $A_1 \cap A_2 \neq \varnothing$, then prove that $A_1 \cup A_2$ is also connected.

(c) Define Totally Bounded Set in Metric Space. Prove that a totally bounded set in the metric space $\mathbb{R}_d$ is finite.

Q. 4. Answer any Two of the following : (10)

(a) Give an example of a metric space; which is not complete. Prove that $\mathbb{R}^2$ is a complete metric space; under the usual metric.

(b) Prove that a closed subset of a complete metric space is complete.

(c) Define Contraction Mapping. Prove that every contraction mapping on any metric space is continuous.

Q. 5. Answer any Two of the following : (10)

(a) Define Compact Metric Space. If the metric space M is compact, then prove that every sequence of points in M has a subsequence converging to a point in M.

(b) Prove that a compact subset of any metric space is closed.

(c) Give an example of a family of subsets of the metric space $\mathbb{R}^1$ having the Finite - Intersection property. Let M be a compact metric space and $\mathfrak{I}$ be a family of closed subsets of M having the Finite - Intersection property. Prove that $\bigcap_{F \in \mathfrak{I}} F \neq \phi$.
